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## 18. RL circuits. Current buildup and shutdown

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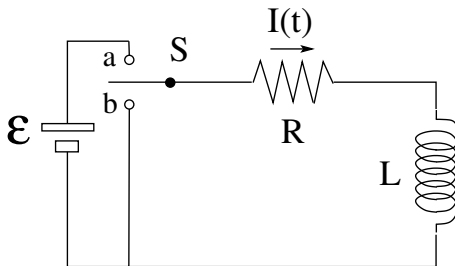


Specifications:

- $\mathcal{E}$  (emf)
- $R$  (resistance)
- $L$  (inductance)

Switch  $S$ :

- a: current buildup
- b: current shutdown



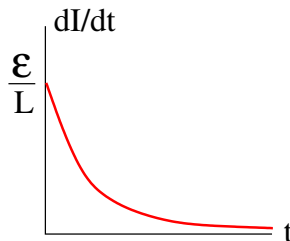
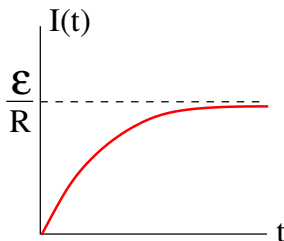
Time-dependent quantities:

- $I(t)$ : instantaneous current through inductor
- $\frac{dI}{dt}$ : rate of change of instantaneous current
- $V_R(t) = I(t)R$ : instantaneous voltage across resistor
- $V_L(t) = L \frac{dI}{dt}$ : instantaneous voltage across inductor

# RL Circuit: Current Buildup in Inductor

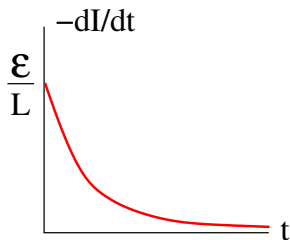
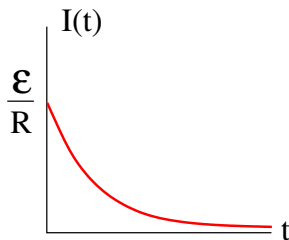


- Loop rule:  $\mathcal{E} - IR - L \frac{dI}{dt} = 0$
- Differential equation:  $L \frac{dI}{dt} = \mathcal{E} - IR \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}/R - I}{L/R}$   
$$\int_0^I \frac{dI}{\mathcal{E}/R - I} = \int_0^t \frac{dt}{L/R} \Rightarrow -\ln\left(\frac{\mathcal{E}/R - I}{\mathcal{E}/R}\right) = \frac{t}{L/R} \Rightarrow \frac{\mathcal{E}/R - I}{\mathcal{E}/R} = e^{-Rt/L}$$
- Current through inductor:  $I(t) = \frac{\mathcal{E}}{R} [1 - e^{-Rt/L}]$
- Rate of current change:  $\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-Rt/L}$





- Loop rule:  $-IR - L \frac{dI}{dt} = 0$
- Differential equation:  $L \frac{dI}{dt} + IR = 0 \Rightarrow \frac{dI}{dt} = -\frac{R}{L} I$   
 $\Rightarrow \int_{\mathcal{E}/R}^I \frac{dI}{I} = -\frac{R}{L} \int_0^t dt \Rightarrow \ln \frac{I}{\mathcal{E}/R} = -\frac{R}{L} t \Rightarrow \frac{I}{\mathcal{E}/R} = e^{-Rt/L}$
- Current:  $I(t) = \frac{\mathcal{E}}{R} e^{-Rt/L}$
- Rate of current change:  $\frac{dI}{dt} = -\frac{\mathcal{E}}{L} e^{-Rt/L}$



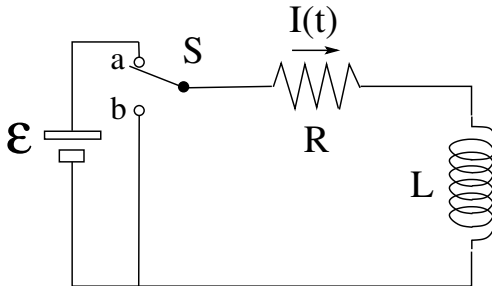
# RL Circuit: Energy Transfer During Current Buildup



Loop rule:  $IR + L\frac{dI}{dt} = \mathcal{E}$  ( $I > 0$ ,  $\frac{dI}{dt} > 0$ )

- $I\mathcal{E}$ : rate at which EMF source delivers energy
- $IV_R = I^2R$ : rate at which energy is dissipated in resistor
- $IV_L = LI\frac{dI}{dt}$ : rate at which energy is stored in inductor

Balance of energy transfer:  $I^2R + LI\frac{dI}{dt} = I\mathcal{E}$



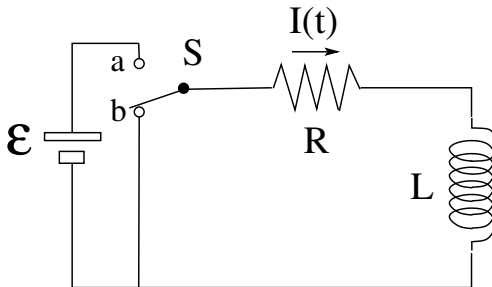
# RL Circuit: Energy Transfer During Current Shutdown



Loop rule:  $IR + L \frac{dI}{dt} = 0$  ( $I > 0$ ,  $\frac{dI}{dt} < 0$ )

- $IV_L = LI \frac{dI}{dt}$ : rate at which inductor releases energy
- $IV_R = I^2 R$ : rate at which energy is dissipated in resistor

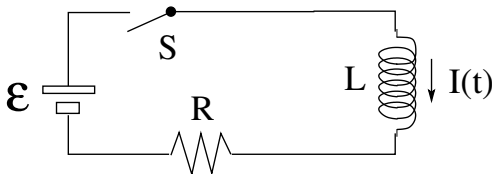
Balance of energy transfer:  $I^2 R + LI \frac{dI}{dt} = 0$





Specification of  $RL$  circuit  
by 3 device properties:

- $\mathcal{E}$  [V] (emf)
- $R$  [ $\Omega$ ] (resistance)
- $L$  [H] (inductance)



Physical properties of  $RL$  circuit during current buildup determined by 3 combinations of the device properties:

- $\frac{\mathcal{E}}{L} = \left. \frac{dI}{dt} \right|_{t=0}$  : initial rate at which current increases
- $\frac{\mathcal{E}}{R} = I(t = \infty)$ : final value of current
- $L/R = \tau$ : time it takes to build up 63% of the current through the circuit  
[ $1 - e^{-1} = 0.632 \dots$ ]

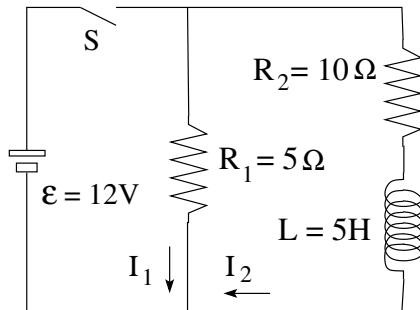
## RL Circuit: Application (8)



In the circuit shown the switch has been open for a long time.

Find the currents  $I_1$  and  $I_2$

- just after the switch has been closed,
- a long time later,
- as functions of time for  $0 < t < \infty$ .

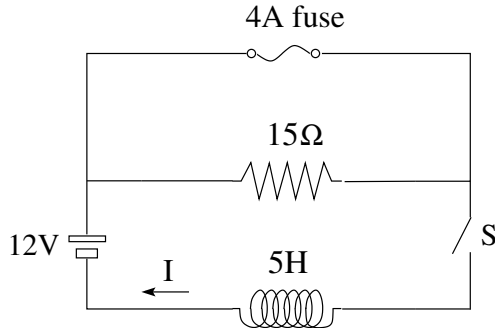






In the circuit shown the switch  $S$  is closed at time  $t = 0$ .

- (a) Find the current  $I$  as a function of time for  $0 < t < t_F$ , where  $t_F$  marks the instant the fuse breaks.
- (b) Find the current  $I$  as a function of time for  $t > t_F$ .

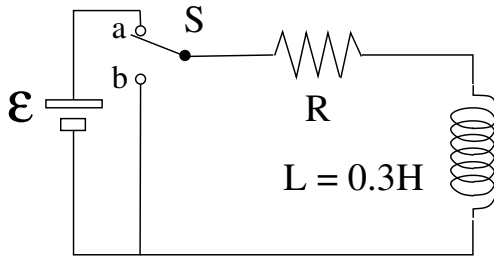


## RL Circuit: Application (6)



In the  $RL$  circuit shown the switch has been at position  $a$  for a long time and is thrown to position  $b$  at time  $t = 0$ . At that instant the current has the value  $I_0 = 0.7\text{A}$  and decreases at the rate  $dI/dt = -360\text{A/s}$ .

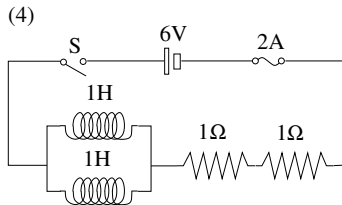
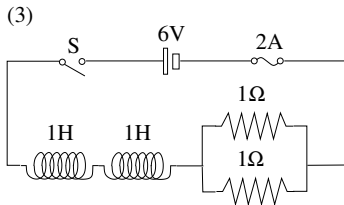
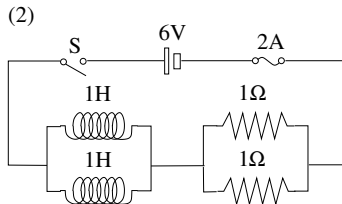
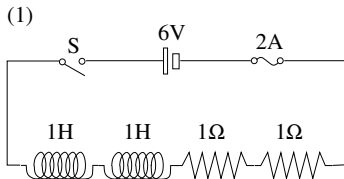
- (a) Find the EMF  $\mathcal{E}$  of the battery.
- (b) Find the resistance  $R$  of the resistor.
- (c) At what time  $t_1$  has the current decreased to the value  $I_1 = 0.2\text{A}$ ?
- (d) Find the voltage across the inductor at time  $t_1$ .





Each  $RL$  circuit contains a 2A fuse. The switches are closed at  $t = 0$ .

- In what sequence are the fuses blown?



## RL Circuit: Application (1)



Each branch in the circuit shown contains a 3A fuse. The switch is closed at time  $t = 0$ .

- (a) Which fuse is blown in the shortest time?
- (b) Which fuse lasts the longest time?

